Cheryl Q. Nelson and Nicole L. Williams

# Sprinklers and Amusement Parks: What Do They Have to Do with Geometry?

Realistic Mathematics Education (RME), a theory of mathematics learning and instruction, was utilized in the development of the following activity. RME incorporates the two principles below, which were synthesized from de Lange and Treffers (Meyer 2001):

1. The starting point of instruction should be experientially real to students, allowing them to engage in meaningful mathematical activity.
2. The learning of a concept passes through various stages of abstraction. The initial stage should be a concrete example in which students can use their informal knowledge to construct their own personal meaning and connections. Students will get to greater abstract levels through various representations such as models, diagrams, and symbolic notation.

This department is designed to provide in reproducible formats activities appropriate for students in grades 7–12. The material may be reproduced by classroom teachers for use in their own classes. Readers who have developed successful classroom activities are encouraged to submit manuscripts, in a format similar to "Activities" already published. Of particular interest are activities focusing on the Council's curriculum standards, its expanded concept of basic skills, problem solving and applications, and the uses of calculators and computers. Send submissions to "Activities" by accessing **mt.msubmit.net**.

Another source of activities can be found in NCTM's *Using Activities from the "Mathematics Teacher" to Support* Principles and Standards, edited by Kimberley Girard and Margaret Aukshun (order number 12746; $35.95), which also includes a grid to help teachers choose the activities that best meet the needs of their students.

*Edited by* ***Susan Beseler***
*Department of Curriculum and Instruction, University of Alabama, Tuscaloosa, AL 35487*

Two tasks presented in this activity were developed in accordance with these principles. Since most students have knowledge of sprinklers and amusement parks, the first principle is satisfied through experientially real activities. The second principle is achieved as students transfer the real-life situation into a geometric model, further analyzing this model to determine the appropriate constructions.

All too often, students in a geometry class learn terms and constructions with no idea of where they would be useful. Two examples are angle bisectors and perpendicular bisectors. Typically, students bisect angles and segments using a straightedge and compass. They then use these concepts to construct the incenter and circumcenter of triangles. Students primarily focus on these concepts in an abstract setting. This activity provides two real-world applications for bisectors that use the students' own intuitions to guide their exploration and discovery. With these applications, we found not only that students understood the geometric concepts better but also that the applications provided a foundation for them to develop a broader sense of the purpose of geometry.

## TEACHER NOTES FOR THE SPRINKLER PROBLEM

Students are confronted with a problem in which a circular sprinkler is to be placed in a triangular park (see **fig. 1** on **sheet 1**). That is, students need to find a center—specifically, the incenter—of the triangle. Students can use a straightedge and compass to accomplish this task; but dynamic software such as The Geometer's Sketchpad (GSP) (Jackiw

2001) is more effective in solving this problem as students can generate the incenter quickly and precisely. Students can then alter the triangle to generalize the solution to any triangle. They can even decide for which type of triangle it would make the most sense to use a circular sprinkler.

***Extension:*** Many extensions are possible for this problem. For example, we could ask, "What percentage of the lawn is not getting water?" "What if the park were in the shape of a square? a rectangle? a diamond?"

## TEACHER NOTES FOR THE AMUSEMENT PARK PROBLEM

Many of us have gone to amusement parks and searched for the closest vendor selling soft drinks. In this application (see **fig. 2** on **sheet 2**), students need to place a soft drink stand so that it is the same distance from the three most popular rides in the park (the vertices of a triangle). With this task, students need to think what it means to be equidistant from the vertices of a triangle and how they can find this point, called the circumcenter of the triangle. When first confronted with this problem, students can explore solutions using either software or a straightedge and compass. If they choose geometric software, however, they can further investigate what happens as the shape of the triangle changes.

***Extension:*** Sometimes the location of a vendor can create difficulties in terms of crowds and long lines. In the early stages of developing a new amusement park, planners should explore where to place the three most popular rides so that a vendor is equidistant from all three but at the same time not within the triangle. What are some of the properties of this triangle?

## REFLECTIONS AND CONCLUSION

This particular activity requires that students analyze the centers of a triangle in order to determine which ones would make sense in solving two real-world problems. If dynamic software is available, potential solutions are more readily tested and verified. As geometry concepts are examined through real-world situations, students make connections to more formal abstract mathematics.

## SOLUTIONS

### *Sheet 1*

1. People walking the sidewalk would get wet. It would waste water.
2. No, the sprinkler cannot reach the corners unless the circle it makes sprays the sidewalks.
3. Answers will vary with students.
4. Answers will vary with students.

***Using GSP to assist in solving the Sprinkler problem***

12. (*a*) The circle is inscribed in the triangle; this means that each side of the triangle is tangent to the circle.

(*b*) No, the circle remains inscribed in the triangle.

(*c*) It would pass through the center of the circle and the vertex of the angle.

(*d*) It shows how to find the center of the circle which is where the park's sprinkler should be located.

### *Sheet 2*

1. The location will provide easy access to the stand and more potential customers will be attracted to it.
2. Students will probably choose a location within the triangle formed.
3. Answers will vary with students.

***Using GSP to assist in solving the Amusement Park problem***

8. (*a*) The circle is circumscribed about the triangle. This means that every vertex of the triangle is on the circle.

(*b*) No, the circle remains circumscribed about the triangle.

(*c*) It would pass through the center of the circle and the midpoint of the side.

(*d*) It shows how to find the point that is the same distance to each of the three rides at the park.

## BIBLIOGRAPHY

Jackiw, Nicholas. The Geometer's Sketchpad. Berkeley, CA: Key Curriculum Press, 2001.

Meyer, Margaret R. "Representation in Realistic Mathematics Education." In *The Roles of Representation in School Mathematics,* edited by A. A. Cuoco. Reston, VA: National Council of Teachers of Mathematics, 2001.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000. ∞


CHERYL NELSON, cnelson@winona.edu, is an associate professor of mathematics education, Department of Mathematics and Statistics, at Winona State University, Winona, MN 55987. NICOLE WILLIAMS, nwilliams@winona.edu, is an assistant professor of mathematics education, Department of Mathematics and Statistics, Winona State University.

# Bisectors in Geometry



### *The Sprinkler Problem*

The Parks Department is installing a circular sprinkler (a sprinkler whose spray makes a perfect circle) to water the lawn at a park that is in the shape of a triangle and is surrounded by sidewalks (see **fig. 1**). The sprinkler should be placed so as to cover as much lawn as possible without spraying the sidewalks.

1. Why wouldn't you want water to spray the sidewalks?

2. Will all of the lawn be watered? Why or why not?

3. Indicate on **figure 1** where you think the sprinkler should be placed.

4. Draw a rough sketch of the circle that the spray from the sprinkler would make.

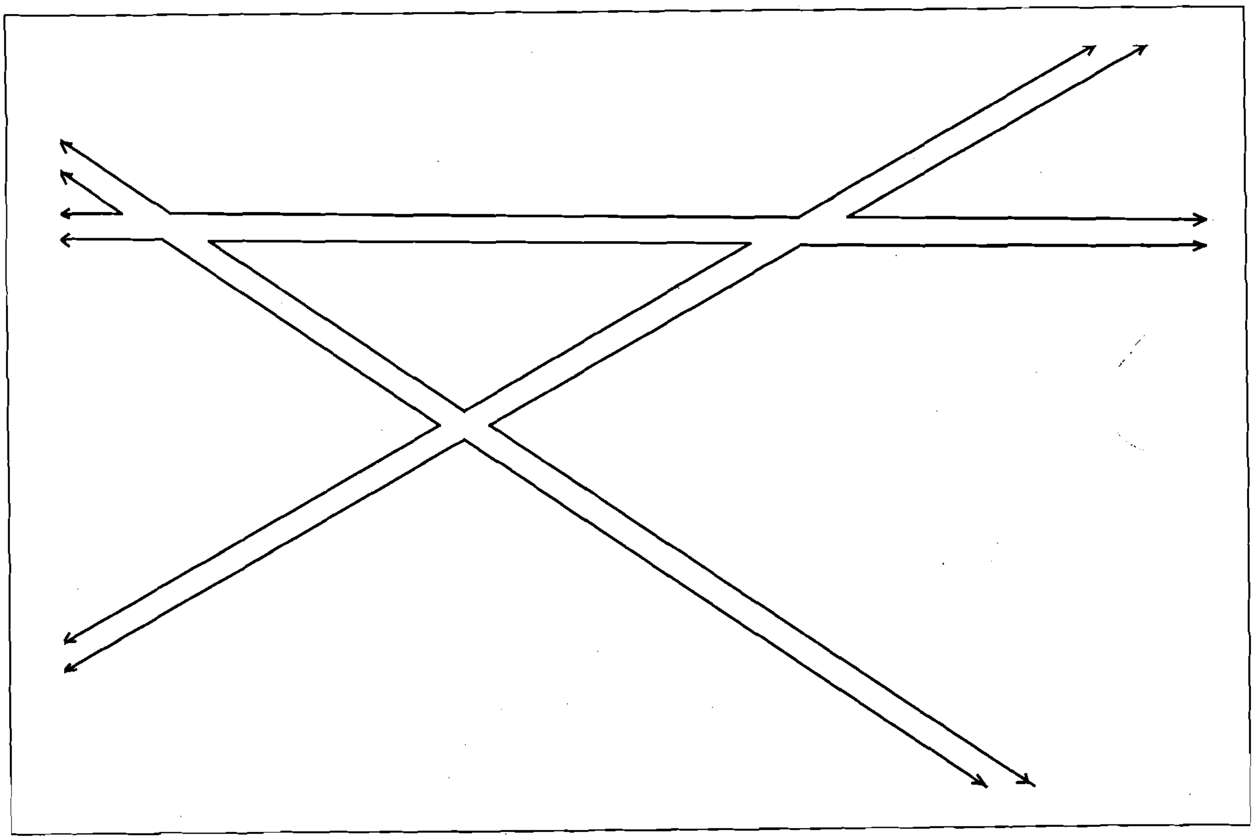

**Fig. 1** Triangular park

### *The Sprinkler Problem*

The Geometer's Sketchpad can assist in solving this problem.

***How to use GSP to assist in solving the Sprinkler problem***

5. Construct a triangle.
6. Construct the bisectors of two angles of the triangle.
7. Construct a point at the intersection of the angle bisectors. This point is one of the centers of the triangle and is called the **incenter** of the triangle (point $G$ in **figure 1.1**).
8. Construct a perpendicular line from the incenter to one side of the triangle.
9. Construct the intersection of the perpendicular line and the side of the triangle.
10. Create the segment from the center of the circle to this intersection. This segment can be used as the radius of a circle centered at the incenter of the triangle.
11. Using the incenter and the radius, construct the circle.
12. Locate the three points where the circle and the triangle intersect. Your construction should look like **figure 1.1**.

    (*a*) Describe the relationship between the triangle and the circle.

    (*b*) Move the vertices of the triangle to change its shape and size. Does this affect the relationship between the circle and the triangle?

    (*c*) If the bisector of the third angle of the triangle were constructed, state two points that it would pass through.

    (*d*) Explain how this construction helps you solve the worksheet problem.

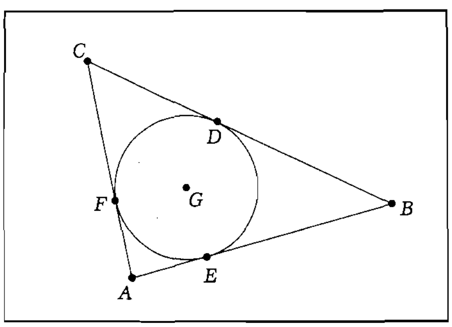


**Fig. 1.1** Model of the incenter of a triangle

*The Amusement Park Problem*

Jo wants to open a new soft drink stand at an amusement park. She looks at the map of the amusement park (**fig. 2**) and notes the three most popular rides: roller coaster, swings, and bumper cars. Jo decides to locate the stand so that it is the same distance from all three of these rides.

1. Why do you think that Jo wants the stand located at a point equidistant from the three most popular rides?

2. Indicate on **figure 2** where you think the stand should be placed.

3. Do you think this location is a good place for a stand? Explain.

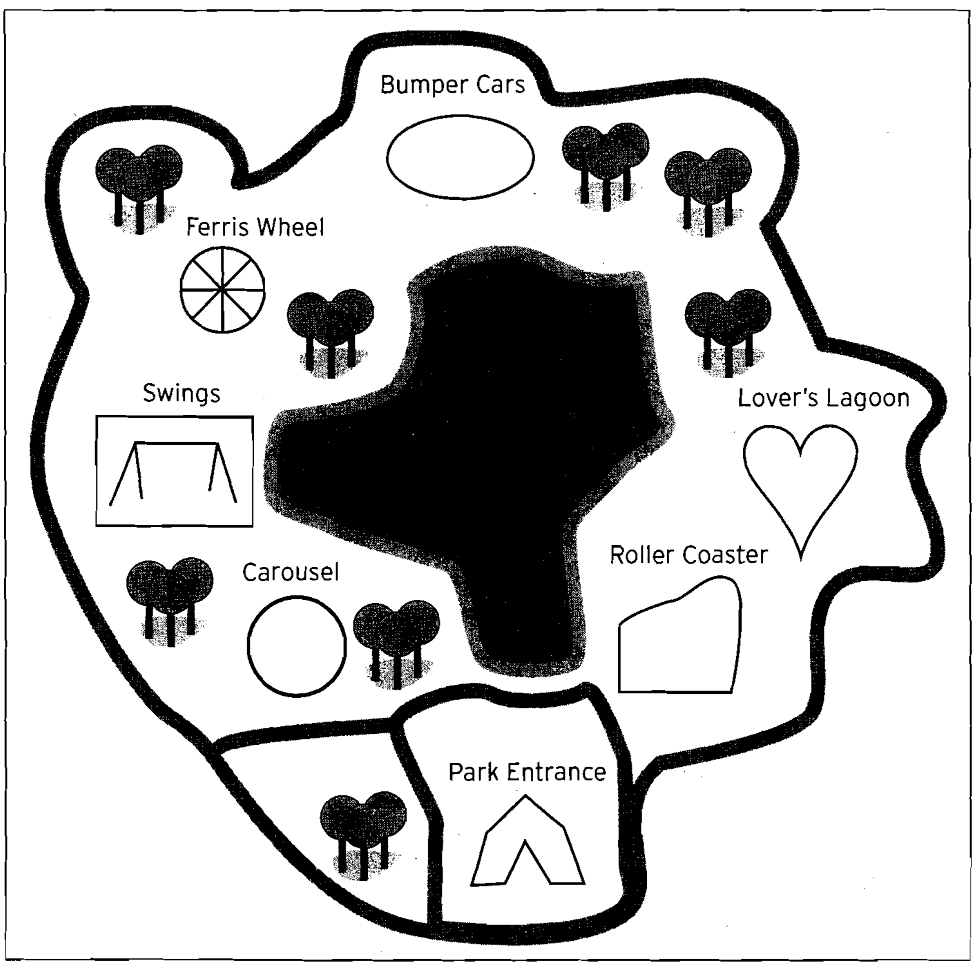


**Fig. 2** Amusement park

*The Amusement Park Problem*

The Geometer's Sketchpad can assist in solving this problem.

***How to Use GSP to assist in solving the Amusement Park problem.***

4. Construct a triangle.
5. Choose any two sides of the triangle and construct the perpendicular bisector of each.
6. Construct a point at the intersection of the two perpendicular bisectors. This point will become the center of a circle and is called the circumcenter of the triangle.
7. Draw a segment from the intersection point to one of the vertices of the triangle. This segment will be the radius of a circle.
8. Construct a circle that has the intersection of the perpendicular bisectors as its center and the segment created above as its radius. Your construction should look like **figure 2.1**.
   (*a*) Describe the relationship between the triangle and the circle.

   (*b*) Move the vertices of the triangle to change its shape and size. Does this affect the relationship between the circle and the triangle?

   (*c*) If the perpendicular bisector of the third side of the triangle were constructed, state two points that it would pass through.

   (*d*) Explain how this construction helps you solve the worksheet problem.

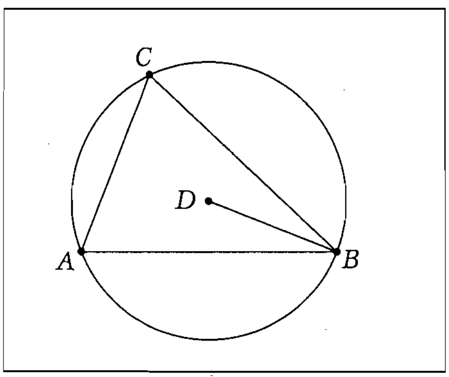


**Fig. 2.1** Model of the circumcenter of a triangle